# Chapter C9: Rotational Energy

This chapter is about how to account for energy when things are spinning. There's not really any new physics in this chapter. The main idea is that from the definition of angle, Eq. (1) below, we can figure out how to expand our energy bookkeeping to keep track of the energy of rotating objects.

## C9.3: Measuring Angles

You will use radians and you'll like them. Really.

An angle $\theta$ is defined by:

$$|\theta| \equiv \frac{s}{r} \, . \tag{1}$$

This formula gives the angle in *radians*, not degrees.

## C9.4: Angular Velocity

Definitions and Equations:

$$\omega \equiv \left| \frac{d\theta}{dt} \right| \, . \tag{2}$$

Since $s = r\theta$,

$$ds = r|d\theta| \, . \tag{3}$$

Thus,

$$v = \frac{ds}{dt} = \frac{r|d\theta|}{dt} = r\omega \, . \tag{4}$$

**Example:**

What is the angular velocity of the minute hand on the clock?
If the minute hand is 25 cm long, what is the speed of the end of the minute hand?

## C9.5: The moment of inertia

Equation C9.7 gives the central idea of this chapter:

$$K^{\text{rot}} = \frac{1}{2} I \omega^2 \,, \tag{5}$$

where $I$, the *moment of inertia*, is given by:

$$I = \sum_{i=1}^{N} m_i r_i^2 \,. \tag{6}$$

## C9.7: Translation and Rotation

The total kinetic energy of an object that is rotating and moving is given by:

$$K = K^{\text{cm}} + K^{\text{rot}} = \frac{1}{2} M v_{\text{cm}}^2 + \frac{1}{2} I \omega^2 \,, \tag{7}$$

where:

## C9.8: Rolling without Slipping

If an object rolls without slipping at a velocity $v_{\text{cm}}$, then:

$$\omega = \frac{v_{\text{cm}}}{R} \,. \tag{8}$$

**Example:** A .5kg plate rolls without slipping at 1 m/s. What is the plate's total kinetic energy?

**Practice:**

1. A DJ plays a record; the turntable revolves at 33.3 revolutions per minute.
   (a) What is the angular velocity of the vinyl?
   (b) What is the speed of a point on the record 7 cm from the center of the record?
   (c) The record has a mass of .2 kg. What is its rotational kinetic energy?

2. A hollow sphere of radius 3m and mass 300 kg is rotating in deep space at 10 revolutions per second around its axis. What it its rotational kinetic energy?

3. A .5 kg lacrosse ball is rolling toward you at 5 m/s. The radius of the ball is 4 cm. What is the total kinetic energy of the ball?