

## Chapter C12: Power!

### C12.1: Power

Power in physics is defined as rate of energy transfer—energy per time. The unit of power is the *Watt*;

$$1\text{Watt} \equiv 1\text{J/s} \quad (1)$$

Power companies measure energy in units of *kilowatt hours*

$$1\text{kW} \cdot \text{h} = 3.6\text{MJ} . \quad (2)$$

### Examples:

1. You put 100 grams of ice in one kilogram of pure alcohol. What is the final temperature of the mixture? Assume that the ice is  $-10$  when it is placed in the alcohol and that the alcohol is at 25 degrees before it is cooled down.
2. You need a heater that can raise the temperature of water  $30^\circ\text{C}$  in 15 minutes. What power must the heater be capable of delivering?
3. How much does it cost in Maine to run a toaster for 5 minutes?

## Some handy info

- Conversion Factors:

$$1\text{kW} \cdot \text{hr} = 3,413 \text{ BTU} . \quad (3)$$

$$1\text{kW} \cdot \text{hr} = 3.6 \text{ MJ} . \quad (4)$$

$$746 \text{ Watts} = 1 \text{ horsepower} . \quad (5)$$

$$1 \text{ Barrel of oil} = 5.74 \times 10^9 \text{ J} . \quad (6)$$

$$1 \text{ Ton of oil} = 4.19 \times 10^{10} \text{ J} . \quad (7)$$

$$1 \text{ Liter of gasoline} = 3.2 \times 10^7 \text{ J} . \quad (8)$$

- A typical shower head dispenses 4 gallons of water per minute.
- A top-loading washing machine uses around 35-50 gallons per wash.
- An electric dryer draws around 3 kilowatts.
- A hair dryer draws around 1500 Watts.
- A toaster draws around 1500 Watts.
- The intensity of direct sunlight on the surface of the earth is 1000 Watts per square meter.
- Typical solar panels are around 10% efficient.
- Electrical power is given by current  $I$  times voltage  $V$ . If  $V$  is measured in volts and  $I$  in Amps, then  $P$  has units of Watts.
- The voltage in U.S. homes is 120 V.
- The solar energy incident on Portland, ME in January is, on average 3.9 kiloWatt hours per square meter per day.
- Residential electric power in Maine is sold by Bangor Hydro for around 16 cents per kiloWatt hour.

## Practice

1. What is the minimum cost of bringing 1 kettle of cold tap water to a boil?
2. A small motor is used to power a lift that raises a 50 kg crate of tofu to a height of 5 meters in 10 seconds. What is the minimum power that the motor must provide?
3. A 55 kg person bikes up Cadillac mountain in 20 minutes. What is the minimum power they must exert? Express your answer in Watts and horsepower.
4. A one-foot length of pipe with a diameter of 1 cm freezes in your basement. You plan on melting the ice in the pipe by heating it with a hair dryer. What is the minimum amount of time it will take to melt the ice?
5. You prop open the door of your refrigerator. Will the room get cold, get hot, or stay the same temperature?
6. A 1000 kg car drives up a 10 % incline at 20 m/s. (A 10 percent grade means that for every 10 meter traveled horizontally the gain in elevation is 1 meter.) What is the minimum horsepower needed for the car to do this, given that the car is about 15 % efficient?
7. You are thinking about doing some work on your house, with the goal of making it more energy efficient and thus saving money. But your budget is limited, so you can't do everything you'd like to do. Imagine you're trying to decide between a new hot water heater and a new furnace. As part of that decision-making process, you might need to estimate how much it costs to heat the water in your home in one month. We'll do the estimation just for showers; assume there are two adults in your household.
  - (a) How many gallons of water do you use per month showering?
  - (b) How much of this is hot water?
  - (c) What temperature do you think the hot water is? What temperature do you think the cold water is entering the tank?
  - (d) How much energy does it take to heat this amount of water to the hotter temperature?
  - (e) How much would Bangor Hydro charge you for this energy?

## C13.2: Cross Product

The cross product is, like the dot product, a way to “multiply” two vectors together. The dot product takes two vectors and turns them into a scalar. The cross product takes two vectors and returns another vector.

$$\text{mag}(\vec{u} \times \vec{w}) = uw \sin \theta \quad (9)$$

The direction of  $\vec{u} \times \vec{w}$  is perpendicular to the plane that contains  $\vec{u}$  and  $\vec{w}$  and is given by the right hand rule.

In components:

$$\vec{u} \times \vec{w} \equiv \begin{bmatrix} u_y w_z - u_z w_y \\ u_z w_x - u_x w_z \\ u_x w_y - u_y w_x \end{bmatrix} \quad (10)$$

We won't use this equation explicitly, but it is perhaps comforting to know that it exists.

Note that  $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$ .

### Examples

- Let  $\vec{u}$  be a displacement vector of 2 meters that points due east, and let  $\vec{w}$  be a vector with a magnitude of 3 meters that points due south.
  - Find  $\vec{u} \times \vec{w}$ .
  - Find  $\vec{u} \cdot \vec{w}$ .
  - Find  $\vec{u} \cdot \vec{u}$ .
  - Find  $\vec{u} \times \vec{u}$ .
- Let  $\vec{v}_1$  be a displacement vector of 3 meters that points due east, and let  $\vec{v}_2$  be a vector with a magnitude of 2 meters that points 45 degrees north of west.
  - Find  $\vec{v}_1 \times \vec{v}_2$ .
  - Find  $\vec{v}_1 \cdot \vec{v}_2$ .
- Let  $\vec{a}$  be a displacement vector of 3 meters that points due east, and let  $\vec{b}$  be a vector with a magnitude of 2 meters that points due west.
  - Find  $\vec{a} \times \vec{b}$ .
  - Find  $\vec{a} \cdot \vec{b}$ .