C8: Force and Energy

This chapter is about *relationships...*

C8.1: Momentum and Kinetic Energy

Kinetic energy is related to momentum:

$$K = \frac{p^2}{2m} . \tag{1}$$

Not all momentum transfers lead to a change in kinetic energy. Consider a small momentum transfer $d\vec{p}$. Suppose an object is moving at velocity $\vec{v}$. Only the "portion" of $d\vec{p}$ that is in the same direction as $\vec{v}$ will lead to a change in kinetic energy.

$$dK = v dp \cos\theta = \vec{v} \cdot d\vec{p} , \tag{2}$$

where v is the speed of the object, dp the magnitude of the momentum transfer, and θ is the angle between $\vec{v}$ and $\vec{dp}$.

C8.2: The Dot Product

The dot product between $\vec{u}$ and $\vec{v}$ is the magnitude of $\vec{u}$ times that portion of mag($\vec{u}$) that's in $\vec{u}$'s direction.

Two important formulas:

$$\vec{u} \cdot \vec{w} \equiv uv\cos\theta , \tag{3}$$

where $\theta \equiv$ the angle between $\vec{u}$ and $\vec{v}$. Also,

$$\vec{u} \cdot \vec{w} = u_x w_x + u_y w_y + u_z w_z . \tag{4}$$

Note that:

- $\vec{u} \cdot \vec{w}$ is a scalar.
- $\vec{u} \cdot \vec{w}$ can be positive or negative.
- $\vec{u} \cdot \vec{u} = u^2$.

C8.3 An Interaction's Contribution to dK

An interaction gives rise to a force on an object. The amount by which this interaction changes the object's kinetic energy is given by:

$$[dK] \equiv \vec{F} \cdot d\vec{r} \tag{5}$$

Recall that Force and impulse $[d\vec{p}]$ are related by:

$$\vec{F} \equiv \frac{[d\vec{p}]}{dt} \,. \tag{6}$$

C8.4 The Meaning of k-Work

When there's a kinetic energy transfer $[dK]$, the energy comes from some sort of potential energy—it does not come from another interaction. Remember that potential energy is a property of an interaction, not a property of a particular object.

C8.5 The Earth's Kinetic Energy

Yet again, we note that the earth is way bigger than us.

C8.6 Force Laws

Don't worry about this section. The main point is that one can go from a potential energy function to a force and vice-versa.

C8.7 Contact Interactions

The normal (perpendicular) part of a contact interaction contributes no k-work.

Examples

1. Consider the following three vectors: $\vec{a} = 3$ m, due North; $\vec{b} = 2$ m at 37 degrees North of East; $\vec{c} = 4$ m due South. Calculate:

   (a) $\vec{a} \cdot \vec{b}$
   (b) $\vec{b} \cdot \vec{c}$
   (c) $\vec{a} \cdot \vec{c}$

Practice

1. You want a spring that is capable of launching a full Nalgene bottle around 1 meter into the air if the spring is compressed by 5 cm. What must the spring constant be for such a spring?

2. Consider two displacement vectors: $\vec{v_1} = [2m, -4m]$ and $\vec{v_2} = [3m, -1m]$. Calculate $\vec{v_1} \cdot \vec{v_2}$. Calculate the angle between $\vec{v_1}$ and $\vec{v_2}$.

3. A 0.5 kg TAB mug is traveling due north at 10 m/s.

   (a) The object is briefly acted upon by a force of 2 Newtons due east.
   (b) The object is briefly acted upon by a force of 2 Newtons due south.
   (c) The object is briefly acted upon by a force of 2 Newtons 37 degrees west of north.

   In each instance, the force acts on the mug for 1 second. For each force:

   (a) What is the impulse delivered to the mug?
   (b) What is the magnitude of the impulse delivered to the mug?
   (c) What is the k-work given to the mug? I.e., what is its change in kinetic energy?

4. A 2000 kg car rolls down a 37 degree incline at a constant speed of 20 m/s.

   (a) In one second, what energy transfer does the gravitational interaction give to the car?
   (b) Where does this energy transfer go?

5. A car goes over the crest of a hill at 20 m/s. The car then coasts to the bottom of the hill, 50 meters below. Ignoring friction, what is the car's speed at the bottom?