

Chapter C9: Rotational Energy

This chapter is about how to account for energy when things are spinning. There's not really any new physics in this chapter. The main idea is that from the definition of angle, Eq. (1) below, we can figure out how to expand our energy bookkeeping to conveniently keep track of the energy of rotating objects.

C9.1: Introduction to Rotational Energy

C9.2: Measuring Angles

You will use radians and you'll like them. Really.

An angle θ is defined by:

$$|\theta| \equiv \frac{s}{r}. \quad (1)$$

This formula gives the angle in *radians*, not degrees.

C9.3: Angular Velocity

Definitions and Equations:

$$\omega \equiv \left| \frac{d\theta}{dt} \right|. \quad (2)$$

Since $s = r\theta$,

$$ds = r|d\theta|. \quad (3)$$

Thus,

$$v = \frac{ds}{dt} = \frac{r|d\theta|}{dt} = r\omega. \quad (4)$$

Example:

What is the angular velocity of the second hand on the clock?

If the second hand is 25 cm long, what is the speed of the end of the hand?

C9.4: The moment of inertia

Equation C9.7 gives the central idea of this chapter:

$$K^{\text{rot}} = \frac{1}{2}I\omega^2, \quad (5)$$

where I , the *moment of inertia*, is given by:

$$I \equiv \sum_{i=1}^N m_i r_i^2. \quad (6)$$

C9.5: Calculating Moments of Inertia

Don't worry about this section. The main idea is that we can use Eq. (6) to determine the moments of inertia of various objects.

C9.6: Translation and Rotation

The total kinetic energy of an object that is rotating and moving is given by:

$$K = K^{\text{cm}} + K^{\text{rot}} = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I\omega^2, \quad (7)$$

C9.7: Rolling without Slipping

If an object rolls without slipping at a velocity v_{cm} , then:

$$\omega = \frac{v_{\text{cm}}}{R}. \quad (8)$$

Example: A .5 kg plate with a radius of 10 cm rolls without slipping at 1 m/s. What is the plate's total kinetic energy?

Practice:

1. A DJ spins a record. The turntable revolves at 33.3 revolutions per minute.
 - (a) What is the angular velocity of the vinyl?
 - (b) What is the speed of a point on the record 7 cm from the center of the record?
 - (c) The record has a mass of .2 kg and a radius of 14 cm. What is its rotational kinetic energy?

2. A hollow sphere of radius 3m and mass 300 kg is rotating in deep space at 10 revolutions per second around its axis. What is its rotational kinetic energy?

3. A .5 kg lacrosse ball is rolling toward you at 5 m/s. The radius of the ball is 4 cm. What is the total kinetic energy of the ball?

4. A solid sphere rolls without slipping down an incline. Initially the sphere is at rest and is 2 meters above the ground. What is the sphere's speed at the bottom of the incline?