Chapter C12: Power!

C12.1: Power

Power in physics is defined as rate of energy transfer—energy per time.

$$\text{Power} \equiv \frac{|\Delta\text{Energy}|}{\Delta\text{time}} \ . \tag{1}$$

The unit of power is the *Watt*;

$$1\text{Watt} \equiv 1\text{J/s} \tag{2}$$

Power companies measure energy in units of *kilowatt hours*

$$1\text{kWh} \ = \ 3.6\text{MJ} \ . \tag{3}$$

Examples:

1. You put 100 grams of ice in one kilogram of pure alcohol. What is the final temperature of the mixture? Assume that the ice is -10 when it is placed in the alcohol and that the alcohol is at 25 degrees before it is cooled down.
2. You need a heater that can raise the temperature of water 30° C in 15 minutes. What power must the heater be capable of delivering?
3. How much does it cost in Maine to run a toaster for 5 minutes?

Some handy info

- Conversion Factors:

$$1\text{kWh} \ = \ 3.6 \text{ MJ} \ . \tag{4}$$

$$746 \text{ Watts} \ = \ 1 \text{ horsepower} \ . \tag{5}$$

- An electric dryer draws around 3 kilowatts.
- A hair dryer draws around 1500 Watts.
- A toaster draws around 1000 Watts.
- A kWh of electrical energy costs \$0.17 in Maine.
- A typical Maine home uses around 400 kWh of electricity a month.
- A typical solar cell in Maine can generate around 10W of electrical power per square meter of solar cell.

Practice

1. What is the minimum cost of bringing 1 kettle of cold tap water to a boil?
2. A small motor is used to power a lift that raises a 50 kg crate of tofu to a height of 5 meters in 10 seconds. What is the minimum power that the motor must provide?
3. A 55 kg person bikes up Cadillac mountain in 20 minutes. What is the minimum power they must exert? Express you answer in Watts and horsepower.
4. A one-foot length of pipe with a radius of 1 cm freezes in your basement. You plan on melting the ice in the pipe by heating it with a hair dryer. What is the minimum amount of time it will take to melt the ice?
5. You prop open the door of your refrigerator. Will the room get cold, get hot, or stay the same temperature?
6. A 1000 kg car drives up a 10 % incline at 20 m/s. (A 10 percent grade means that for every 10 meter traveled horizontally the gain in elevation is 1 meter.) What is the minimum horsepower needed for the car to do this, given that the car is about 15 % efficient?
7. What power is needed for a typical Maine home. (To calculate this, assume that the home draws energy at an equal rate all month.)
8. What area of solar cells would be needed to provide enough energy for a typical Maine home?
9. Estimate how much it costs to heat the water for a typical shower, assuming that you have an electric hot water heater. Assuming you shower daily, how much would this cost per month?

C13.2: Cross Product

The cross product is, like the dot product, a way to "multiply" two vectors together. The dot product takes two vectors and turns them into a scalar. The cross product takes two vectors and returns another vector.

$$\text{mag}(\vec{u} \times \vec{w}) \;=\; uw \sin\theta \tag{6}$$

The direction of $\vec{u} \times \vec{w}$ is perpendicular to the plane that contains $\vec{u}$ and $\vec{w}$ and is given by the right hand rule.

In components:

$$\vec{u} \times \vec{w} \;\equiv\; \begin{bmatrix} u_y w_z - u_z w_y \\ u_z w_x - u_x w_z \\ u_x w_y - u_y w_x \end{bmatrix} \tag{7}$$

We won't use this equation explicitly, but it is perhaps comforting to know that it exists.

Note that $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$.

Examples

1. Let $\vec{u}$ be a displacement vector of 2 meters that points due east, and let $\vec{w}$ be a vector with a magnitude of 3 meters that points due south.
   (a) Find $\vec{u} \times \vec{w}$.
   (b) Find $\vec{u} \cdot \vec{w}$.
   (c) Find $\vec{u} \cdot \vec{u}$.
   (d) Find $\vec{u} \times \vec{u}$.
2. Let $\vec{v}_1$ be a displacement vector of 3 meters that points due east, and let $\vec{v}_2$ be a vector with a magnitude of 2 meters that points 45 degrees north of west.
   (a) Find $\vec{v}_1 \times \vec{v}_2$.
   (b) Find $\vec{v}_1 \cdot \vec{v}_2$.
3. Let $\vec{a}$ be a displacement vector of 3 meters that points due east, and let $\vec{b}$ be a vector with a magnitude of 2 meters that points due west.
   (a) Find $\vec{a} \times \vec{b}$.
   (b) Find $\vec{a} \cdot \vec{b}$.