Chapter N4: Motion from Forces

N4.1: The Reverse Kinematic Chain

As we saw in the last chapter, velocity is the time derivative of position. And acceleration is the time derivative of velocity:

$$v(t) = \frac{dx(t)}{dt} \text{ and } a(t) = \frac{dv(t)}{dt} . \tag{1}$$

This tells us how to go from position $x(t)$ to acceleration $a(t)$. And Netwon's second law ($\vec{F} = m\vec{a}$) lets us figure out what force caused the motion.

This chapter is about "working backwards." Given a force, we can figure out an object's acceleration. We can then take anti-derivatives to go from acceleration to velocity, and velocity to position.

N4.2: Graphical Derivatives

Example:

1. A bug crawls at a constant speed of 3 m/s. Sketch its speed and position as a function of time.
2. A physics textbook falls straight down at a constant acceleration of 10 m/s^2. Sketch its acceleration, velocity, and position as a function of time.

N4.3: Free Fall in One Dimension

If an object is acted on only by gravity near the surface of the earth (i.e., we ignore drag), then its z-position and velocity as a function of time are given by:

$$v_z(t) = v_0 - gt , \tag{2}$$

$$z(t) = -\frac{1}{2}gt^2 + v_0 t + z_0 , \tag{3}$$

where v_0 and z_0 are its initial velocity and position.

N4.6: Constructing Trajectory Diagrams

Previously we have seen how to construct acceleration arrows given a motion diagram. Here, we do it backwards. We construct a motion diagram, given an acceleration arrow and an initial position and velocity.

Example: A climate negotiator runs off a cliff with an initial horizontal velocity of 5 m/s. She experiences a constant, downward acceleration due to gravity. Construct a motion diagram for her trajectory.

Practice:

1. You drop a TAB mug off a 30 meter cliff. How long does it take the mug to hit the ground?
2. An object starts at rest. It accelerates at 10 m/s^2 for five seconds. It then accelerates at -10 m/s^2 for three seconds. Sketch quantitatively accurate graphs for its acceleration, velocity, and position as a function of time.
3. A net force of 100 Newtons is applied to a 25 kg crate of tofu for 3 seconds. After 3 seconds there is no net force on the box. Sketch the acceleration, velocity, and position of the box.
4. You drop a 17 kg box of tempeh off a building. How high must the building be so that the tempeh falls for at least 5 seconds before hitting the ground?
5. A skydiver jumps out of an airplane. She falls toward the earth, and eventually reaches a constant velocity. For each of the following, sketch a free body diagram and net-force diagram:
   (a) The instant after she jumps out of the plane.
   (b) She's been falling for a little while, but hasn't reached her terminal velocity yet.
   (c) She's falling at her terminal velocity.
6. Make a sketch of the skydiver's y, v, and a vs. t.