Chapter C12: Power

Physics I

College of the Atlantic

C12.1: Power

Power in physics is defined as rate of energy transfer—energy per time.

$$\text{Power} \equiv \frac{|\Delta\text{Energy}|}{\Delta\text{time}} \,. \tag{1}$$

The unit of power is the *Watt*;

$$1\text{Watt} \equiv 1\text{J/s} \tag{2}$$

Power companies measure energy in units of *kilowatt hours*

$$1\text{kWh} = 3.6\text{MJ} \,. \tag{3}$$

One kilowatt hour is the amount of energy that is transferred if one kilowatt of power is delivered for one hour.

Examples:

1. You need a heater that can raise the temperature of water 30° C in 15 minutes. What power must the heater be capable of delivering?
2. How much does it cost in Maine to run 2000 Watt electric heater for 8 hours? How much would this cost per month?

Some handy info

- Conversion Factors:

$$1\text{kWh} = 3.6 \text{ MJ} \,. \tag{4}$$

$$746 \text{ Watts} = 1 \text{ horsepower} \,. \tag{5}$$

- The average US energy consumption: 250 kWh per person per day.
- An electric dryer draws around 3 kilowatts.
- A toaster draws around 1000 Watts.
- A kWh of electrical energy costs $0.185 in Maine.
- A typical Maine home uses around 500 kWh of electricity a month.
- A typical solar cell in Maine generates around 10W of electrical power per m^2 of solar cell.
- Burning a kg of gasoline releases roughly 46 MJ of internal energy.
- Burning a kg of natural gas releases roughly 55 MJ of internal energy.
- For chemical reactions, the energy released per kg ranges between 10 and 100 MJ.
- For nuclear reactions, the energy released per kg ranges between 50 and 500 TJ. ($1\text{TJ} = 1 \times 10^{12}\text{J}$.)
- The *calorie* is a unit of energy defined as the amount of energy needed to raise one gram of water by one degree C. 1 cal = 4.182 J.
- The "calorie" used to measure the energy content of food is actually equal to 1000 calories. Food-content calories are referred to as dietary calories, food calories, and large calories. Sometimes large calories are abbreviated C or Cal.

C13.2: Cross Product

The cross product is, like the dot product, a way to "multiply" two vectors together. The dot product takes two vectors and turns them into a scalar. The cross product takes two vectors and returns another vector.

$$\text{mag}(\vec{u} \times \vec{w}) = uw \sin\theta \tag{6}$$

A more physical/geometric way to think of this is:

$$\text{mag}(\vec{u} \times \vec{w}) = uw_\perp = u_\perp w \; . \tag{7}$$

The direction of $\vec{u} \times \vec{w}$ is perpendicular to the plane that contains $\vec{u}$ and $\vec{w}$ and is given by the right hand rule.

In components:

$$\vec{u} \times \vec{w} \equiv \begin{bmatrix} u_y w_z - u_z w_y \\ u_z w_x - u_x w_z \\ u_x w_y - u_y w_x \end{bmatrix} \tag{8}$$

We won't use this equation explicitly, but it is perhaps comforting to know that it exists. Or maybe not. Anyway...

Note that $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$.

Examples

1. Let $\vec{u}$ be a displacement vector of 2 meters that points due east, and let $\vec{w}$ be a vector with a magnitude of 3 meters that points due south.
   (a) Find $\vec{u} \times \vec{w}$.
   (b) Find $\vec{u} \cdot \vec{w}$.
   (c) Find $\vec{u} \cdot \vec{u}$.
   (d) Find $\vec{u} \times \vec{u}$.
2. Let $\vec{v}_1$ be a displacement vector of 3 meters that points due east, and let $\vec{v}_2$ be a vector with a magnitude of 2 meters that points 45 degrees north of west.
   (a) Find $\vec{v}_1 \times \vec{v}_2$.
   (b) Find $\vec{v}_1 \cdot \vec{v}_2$.
3. Let $\vec{a}$ be a displacement vector of 3 meters that points due east, and let $\vec{b}$ be a vector with a magnitude of 2 meters that points due west.
   (a) Find $\vec{a} \times \vec{b}$.
   (b) Find $\vec{a} \cdot \vec{b}$.