Summary of Unit Six

The Lorentz Transformation

Physics II
Special Relativity

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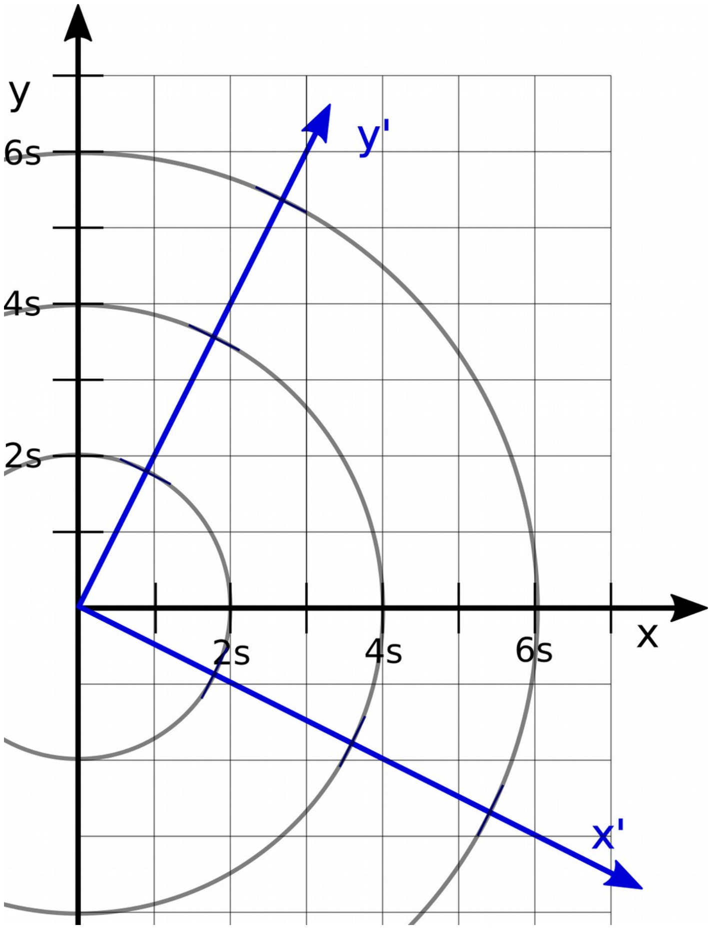
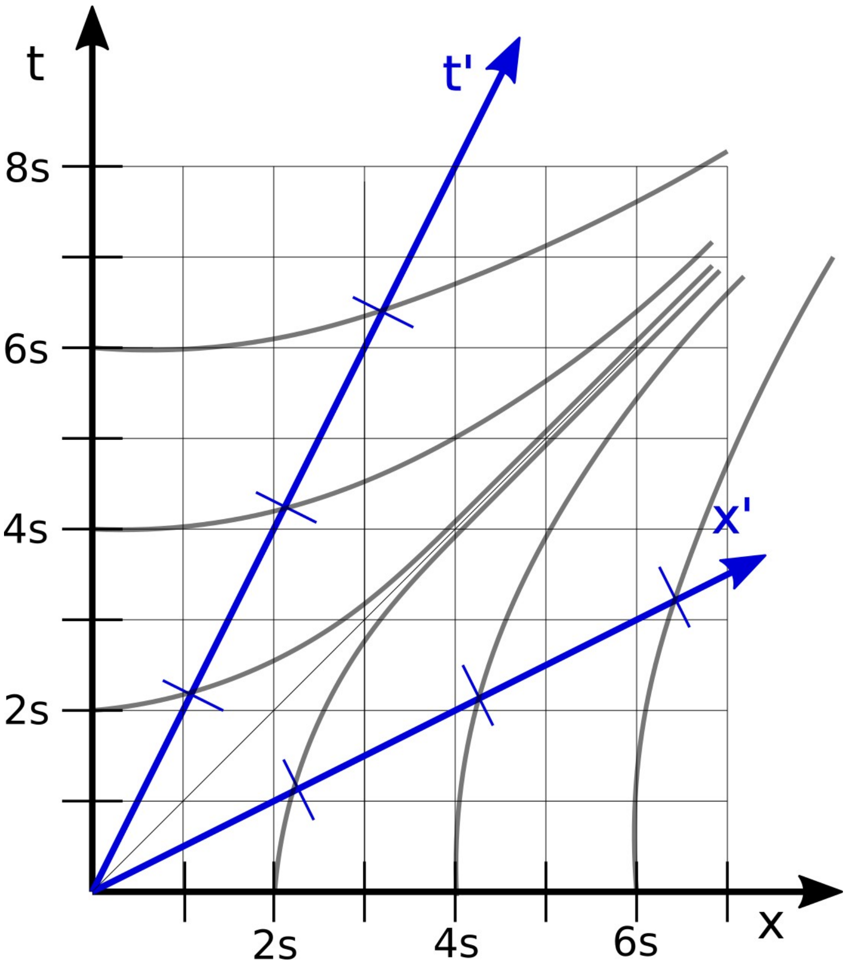
http://tiny.cc/RelativityAtCOA

Two-Observer Coordinate Systems



- A circle is the set of points a constant **distance** from the origin.
- Markings on rotated axes are connected by circles.
- $$d^2 = x^2 + y^2$$

Two-Observer Coordinate Systems



- A hyperbola is the set of points a constant **spacetime interval** from the origin.
- t’ axis has slope 1/β, x’ axis has slope β
- $s^2 = t^2 - x^2$
- Markings on primed axes are connected by hyperbolas.
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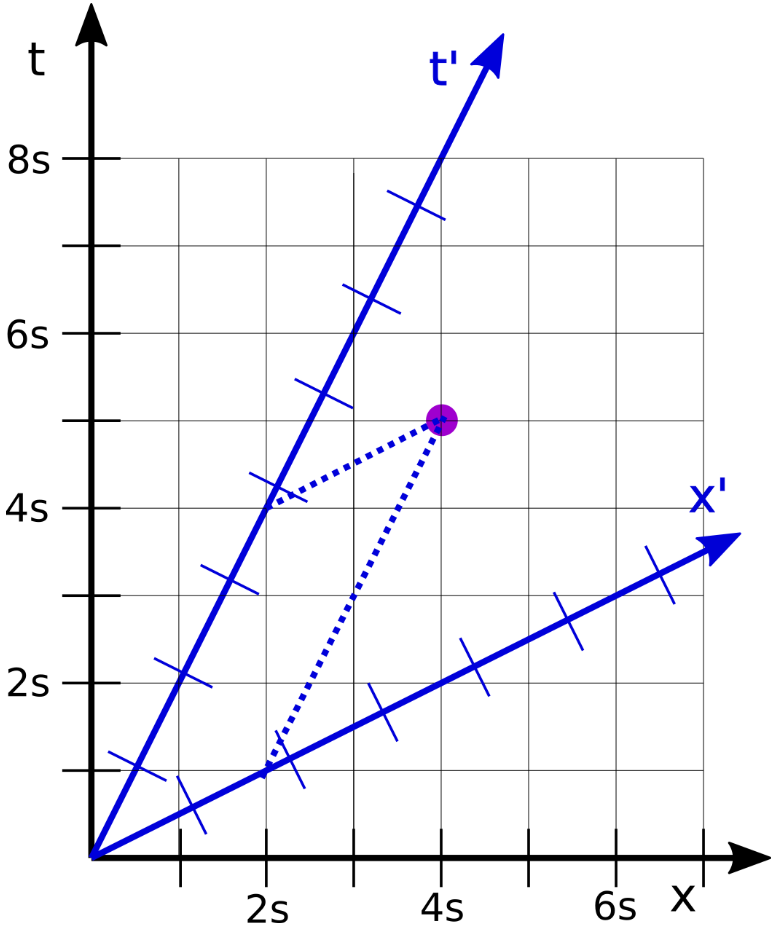
Two-Observer Coordinate Systems



- To calibrate t' axis:
$$\Delta t = \gamma \Delta t' \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$
- x' axis is similar
- Read the primed coordinates via parallel lines.
- Example: $x' \approx 1.6\,, \quad t' \approx 3.8$
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The Lorentz Transformation

$$
\begin{aligned}
t &= \gamma(t' + \beta x') \\
x &= \gamma(\beta t' + x') \\
t' &= \gamma(t - \beta x) \\
x' &= \gamma(-\beta t + x)
\end{aligned}
$$

- Relates space time coordinates in one frame to spacetime coordinates in another frame.
- Relativistic version of the Galilean transformations
- A “dictionary” that lets you translate events from one frame to another.
- We’ve gotten here by assuming that the speed of light is constant in all frames.
- Note: The Lorentz transformation and the two-observer diagram are complementary ways of expressing the same relationship.