Homework 02

Physics II

"Due" Friday, April 8, 2022

College of the Atlantic. Spring 2022

There are two parts to this assignment.

Part 1: WeBWorK. Do Homework 02 which you will find on your WeBWorK page. I recommend doing the WeBWorK part of the homework first. This will enable you to benefit from WeBWorK's instant, if not necessarily friendly, feedback before you do part two.

Part 2: Not WeBWorK. Below are some non-WeBWorK problems.

- If you want, you can do these problems in pairs and hand in only one write-up.
- "Hand in" the problem on google classroom. You can take a picture of your work, or type up your work, or scan your work.

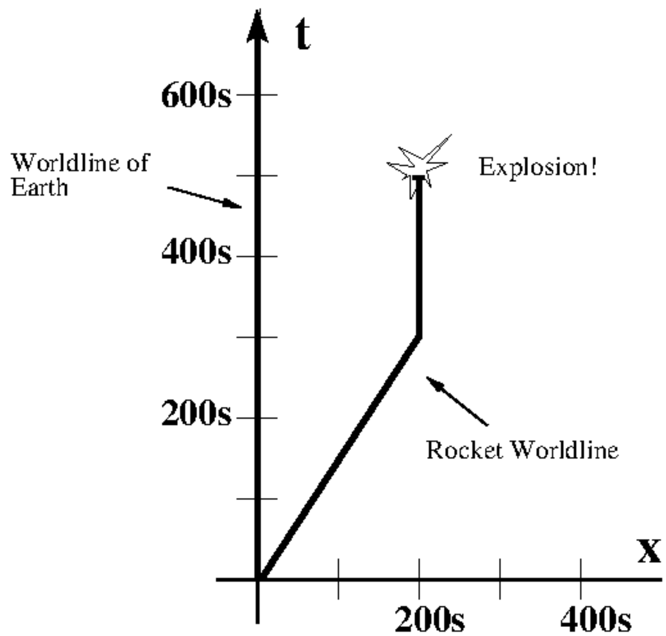

Figure 1: A spacetime diagram.

1. The spacetime diagram in Fig. 1 shows a rocket ship that leaves earth, travels at a constant speed for a while, comes to rest, and then explodes.
   (a) What are the spacetime coordinates of the following events: A, the rocket launches from earth; B, the rocket comes to rest, C, the rocket explodes.
   (b) What is the speed of the rocket before it comes to rest?
   (c) A light signal from earth happens to have reached the rocket at the moment it explodes. Draw the worldline of this light signal on the spacetime diagram. When did the light signal leave earth?

2. Beowulf is in a spaceship traveling toward a space station at a constant speed of $v = 3/4$. Anastajia is on the space station, which we will take to be at $x = 0$ in the space station reference frame. At time $t = 0$ the spaceship is 16 light-hours from the space station. At this time and place, Beowulf sends a pulse of light to Anastajia to signal his intention to land on the space station. Call the launching of this signal event A. The station receives the signal (event B). Anastajia waits half an hour and then sends a laser pulse back to Beowulf, who has continued traveling at $v = 3/4$ all the while. Call Anajstajia's launch of her signal event C. Sometime later the signal reaches Beowulf in his spaceship (event D).

   (a) Draw this situation on a reasonably carefully drawn spacetime diagram. Include the worldlines for the Anastajia and Beowulf. Also include events A, B, C, and D. Use units of light-hours for you axes.

   (b) Exactly where and when does event D occur? Don't just estimate it from your diagram—use math to figure out an exact value.

3. Imagine that an advanced alien race, bent on keeping humans from escaping the solar system, places an opaque spherical force field around the solar system. The force field is 6 light-hours in diameter, is centered on the sun, and is formed in a signal instant of time as measured by synchronized clocks in an inertial frame attached to the sun. This instant corresponds to 9PM on Saturday night in whatever time zone you happen to be in. When does the opaque sphere appear to start blocking light from the stars from your vantage point on earth, 8 light-minutes from the sun? Does the opaque sphere appear all at once? If not, how long does it take for the sphere to appear, and what does it look like as it appears? Describe what you would see as completely as you can. (This is a lightly-edited version of problem R2R.1 from Thomas A. Moore, *Six Ideas that Shaped Physics: Unit R (2nd ed.)*, McGraw Hill, 2003. Moore notes that the inspiration for this problem comes from the novel *Quarantine* by Greg Egan.)

4. At $t = 0$ an alien spaceship passes by the earth. Let this be event A. AT $t = 13$ min (according to synchronized clocks on earth and Mars) the spaceship passes by Mars, which is 5 light-minutes from earth at the time: let this be event B. Radar tracking indicates that the spaceship moves at a constant velocity between earth and Mars. Just after the ship passes earth, people on earth launch a probe whose purpose is to catch up with and investigate the spaceship. This probe accelerates away from earth, moving slowly at first, and moving faster and faster as time passes. Eventually it catches up with and passes the alien ship just as it passes Mars. (Ignore the gravity and relative motion of the earth and Mars, which are small on the scale of minutes. Thus, treat the earth and Mars as if they were both at rest in the inertial reference frame of the solar system. Also, assume that both the probe and the alien spacecraft carry clocks.) (Based on problem R3S.6, Moore, second edition.)

   (a) Draw a *quantitatively* accurate spacetime diagram of the situation, including labeled worldlines for the earth, Mars, the alien spacecraft, and the probe. Also label events A and B.
   (b) Whose clocks measure coordinate times between events A and B? Explain briefly.
   (c) Whose clocks measure proper times between these events. Explain briefly.
   (d) Does any clock in this problem measure the spacetime interval between the events? Explain briefly.

5. A train is moving due east at a large constant speed on a straight track. Imagine that Anastajia is riding on the train exactly in the middle—halfway between the front and the back. Beowulf is sitting by the tracks only a few feet from the train. Let the event of Anastajia's passing Beowulf be the origin O in both frames. At this same instant, both Anastajia and Beowulf receive the light from lighting flashes that have struck both ends of the train. Anastajia concludes that since she is in the middle of the train and she received the light from the strikes at the same time, the lightning strikes must have occurred at the same time in her reference frame. Is she right? If not, which strike really happened first? Can Beowulf conclude from his seeing the flashes at the same time that the strikes happened at the same time in the ground frame? Why or why not? If not, which strike happened first in his frame? (Hint: Draw spacetime diagrams for the situation in both Ana's and Beowulf's frames. Problem R3R.1 from Moore, second edition.)

6. **Optional. Recommended for folks who want to puzzle through some math. This problem basically has you derive the Lorentz equation for time dilation. It's fun. This is a tricky problem, but by no means impossible.** (Based on problem R3A.1 from Moore, second edition.) Consider the figure shown below that we used to derive the result that coordinate time is frame dependent. Let the spatial separation between each side clock and the center clock be $L = 12$ ns. And let the speed of the clocks relative to the at-rest frame be $\beta = 0.4$. Find the time separation $t_B - t_A$ as measured in the at-rest frame. Here are some hints:

   - Work symbolically. Don't plug in numbers for the clock separation L and the speed β until the end.
   - Consider the left clock. In the time between $t = 0$ and $t = t_A$, the clock moves a distance βt_A toward the light flash coming toward it. Thus the total distance that the light flash has to cover in this time interval is not L but instead is $L - \beta t_A$. But since the light flash travels with a speed of 1 in all reference frames, time that the light flash takes to travel to the clock is equal to the *distance* that it has to travel. Write this statement as an equation and solve for t_A.
   - Carry out a similar analysis for t_B.