Blackbody Radiation and Planck's Constant

Physics II: Modern Physics

College of the Atlantic

Blackbody Radiation

How does the the intensity $R(\lambda)$ of radiant energy from a hot object depend on λ, the wavelength of the radiation? Two empirical observations:

1. **Stefan's law:** The total radiant intensity equals σT^4, where σ is the Stefan-Boltzmann constant, and is measured to have a value of

$$\sigma = 5.670 \times 10^{-8} \mathrm{W/m^2K^4} \tag{1}$$

2. **Wien's displacement law:**

$$\lambda_{\mathrm{max}} = \frac{2.898 \times 10^{-3} \mathrm{mK}}{T} , \tag{2}$$

where λ_{max} is the wavelength of the most intense radiation. (It is not the maximum possible wavelength.) Wien's law tells us how the peak of the intensity curve moves—is displaced—as the temperature is varied.

Rayleigh-Jeans law and the Ultraviolet Catastrophe

Using a classical analysis, one can obtain the following result for $R(\lambda)$:

$$R(\lambda) = \frac{8\pi}{\lambda^4} kT \frac{c}{4} . \tag{3}$$

This is known as the Rayleigh-Jeans law. Here, c is the speed of light, and k is Boltzmann's constant; $k = 1.381 \times 10^{-23}$ J/K. Eq. (3) matches experimental results well for large wavelength. The Rayleigh-Jeans law assumes that energy in the cavity, both in the walls of the cavity and in the radiation that is bouncing around the cavity, can assume any value. That is, energy is a continuous variable; it is not quantized.

However, it does not do such a good job for small λ. In fact, we can see that the radiant intensity goes to infinity as λ gets small. Thus, the Rayleigh-Jeans law, if true, would say that all objects are emitting an infinite amount of energy. Clearly this is not the case. This failure of the Rayleigh-Jeans law is known as the *Ultraviolet Catastrophe*.

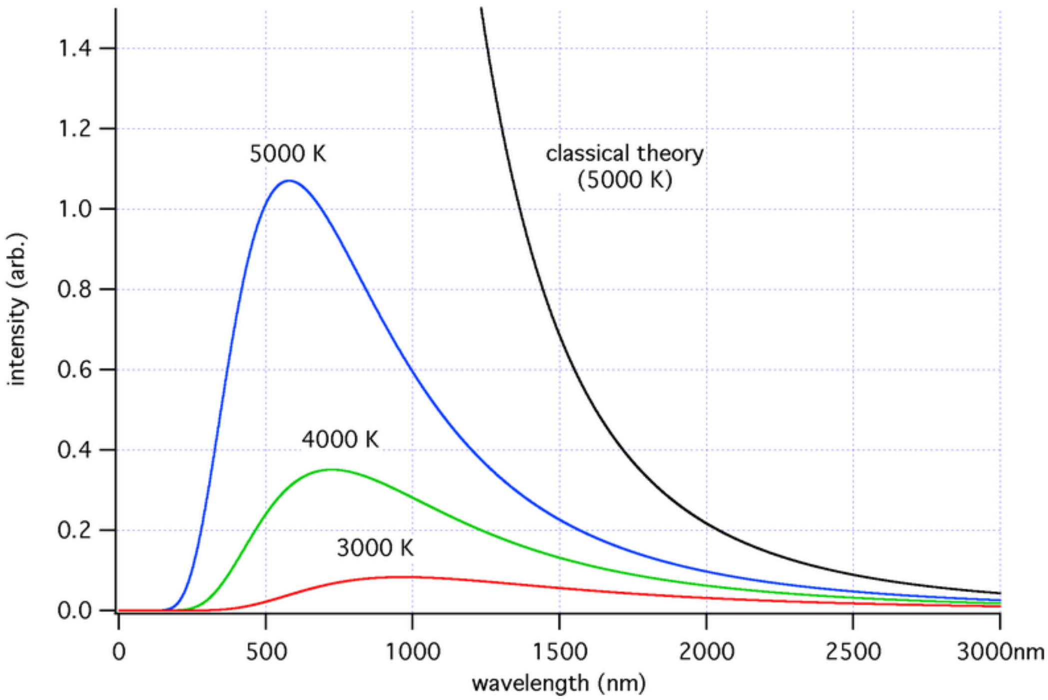

Figure 1: The blackbody emission curves for objects at various temperatures. Also shown is the predicted curve from classical theory. Figure source: `https://en.wikipedia.org/wiki/File:Black_body.svg`, released into the public domain by its creator, Darth Kule.

Planck and his Constant

The walls of the cavity of the box are filled with little oscillators. Electromagnetic radiation is continually being absorbed and re-emitted by these oscillators. Planck said: In an oscillator of frequency ν, the only permitted values of the energy are:

$$E = 0, h\nu, 2h\nu, 3h\nu, \ldots \; . \tag{4}$$

He then was able to derive a better formula than the Rayleigh-Jeans law:

$$R(\lambda) \;=\; \frac{c}{4}\frac{8\pi}{\lambda^4}\left[\frac{hc}{\lambda}\frac{1}{e^{\frac{hc}{\lambda kT}} - 1}\right] \; . \tag{5}$$

This formula fits the experiment data extremely well.

In so doing, Plank introduced a new constant, h, now referred to as *Planck's constant:*

$$h \;=\; 6.62616 \times 10^{-34} \text{ J s} \; . \tag{6}$$