Homework 03

Physics II

“Due” Friday, April 19, 2024

College of the Atlantic. Spring 2024

Part 1: WeBWorK. Do Homework 03 which you will find on your WeBWorK page. I recommend doing the WeBWorK part of the homework first. This will enable you to benefit from WeBWorK’s instant, if not necessarily friendly, feedback before you do part two.

Part 2: Not WeBWorK. Below are some non-WeBWorK problems.

- If you want, you can do these problems in pairs and hand in only one write-up.
- “Hand in” the problem on google classroom. You can take a picture of your work, or type up your work, or scan your work.

1. Based closely on a problem from Tom Moore *Six Ideas that Shaped Physics: Unit R (second edition)*, (2003). In 2095 a message arrives at earth from a growing colony at Tau Ceti, which is 11.3 years from earth. The message asks for help in combating a virus[1] that is making people seriously ill. Using advanced technology available on earth, scientists are quickly able to construct a vaccine that confer immunity to the virus. You have to decide how much of the drug can be sent to Tau Ceti. The space probes available on short notice are capable of doing one of the following:

   - Sending 200 g of the vaccine at a speed of 0.95.
   - Sending 1 kg at a speed of 0.90.
   - Sending 5 kg at a speed of 0.80
   - Sending 20 kg at a speed of 0.6

   The catch is that the vaccine degrades such that after five years it is no longer effective.

   (a) Is it possible to send the vaccine to Tau Ceti? If so, how much can you send?

   (b) What is the slowest speed a spaceship could travel at and still have the vaccine be effective when it arrives on Tau Ceti?

   You’ll save yourself some trial and error if you answer part b first. But you might need to work through a few of the scenarios in part a before you see how to do part b.

2. **Optional.** Recommended. Uses a bit of algebra to motivate the binomial approximation.

   (a) If $x \ll 1$, then $x^2 \ll x$. Convince yourself that this is true with an example. If $x = 0.001$, what is x^2?

   (b) What is $(1 + x)^2$?

   (c) What is $(1+x)^2$ if we ignore x^2 because it is so small? You should have an expression that is equal to the binomial approximation for $a = 2$.

   (d) Repeat the above analysis for $(1 + x)^3$. If you ignore x^2 and x^3 terms, you’ll should get binomial approximation for $a = 3$.

[1] Yes, Tom Moore really has a problem about a serious virus in his 2003 relativity book. A little spooky, eh?

3. **Optional.** Uses a tiny bit of calculus to derive the binomial approximation. Consider the following function:
$$f(x) \;=\; (1+x)^a \;. \tag{1}$$
We would like to approximate this function with a line. I.e., we would like to find the slope m and intercept b such that the line is the best approximation of $f(x)$:
$$f(x) \;\approx\; b + mx \;. \tag{2}$$
   (a) Find b by plugging in $x = 0$ to each side of the equation.
   (b) Take the derivatives of both sides of Eq. (2). Then plug in $x = 0$ and solve for m.
   (c) Plug your b and m values in to Eq. (2). Congratulations. You've derived the binomial approximation. What we've done, as you have likely suspected, is determine the tangent line approximation for $f(x)$ at $x = 0$.

4. **Optional:** Does not require calculus. This is problem R4M.6 from Moore, *Six Ideas that Shaped Physics*, Book R, 3rd ed, McGraw Hill, 2017. The satellites used in the Global Positioning System go around the earth in circular orbits whose radius is $26,600$ km and a whose period is exactly twelve hours. Assume for the sake of simplicity that the earth is not rotating, so that a clock on its surface is in an inertial frame.
   (a) The speed of an object in a circular orbit of radius R around an object with a mass M is $v = \sqrt{GM/R}$, where G is the universal gravitation constant. Argue that in SR units, $G = G_{[SI]}/c^3 = 2.475 \times 10^{-36}$ s/kg. The value of the gravitational constant in SI units is $G_{[SI]} = 6.67 \times 10^{-11}$ N m^2/kg^2.
   (b) Let event A be a certain GPS satellite passing a given position in space and event B be it passing that point again after one complete orbit. At each event, this satellite sends a radio signal to a clock directly below it on the (nonrotating) earth, which receives the signals at events C and D, respectively. What is the difference between the time an atomic clock on board the satellite registers between events A and B and the time a clock on the earth's surface registers between events C and D. Express your results symbolically in terms of G, M, and R. Don't crunch numbers yet. Assume that $GM/R \ll 1$.
   (c) Calculate numerically how much less time a clock on the GPS satellite measures for a complete orbit than the clock on the ground does.

5. **Optional:** This problem and the next are about the path length formula[2] for the length of a curve and are not about relativity. Consider a straight line that goes though the points $(4, 4)$ and $(7, 8)$.
   (a) Use the path length formula.
   (b) Draw a picture of the line and use geometry to figure out the length.

6. **Optional:** Use the path length formula to find the length of the arc of a circle of radius one in the first quadrant. (I.e., find the arc length of a quarter of a unit circle.)

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[2]
$$\text{Path Length} \;=\; \int_{\mathrm{x_a}}^{\mathrm{x_b}} \sqrt{1 + \left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)^2}\, \mathrm{dx}\;.$$