Physics III
Homework Five

Due Friday 27 May, 2007.

Consider the following complex numbers:

$$
\begin{aligned}
z_1 &= 1 - i \; , \\
z_2 &= -2 \; , \\
z_3 &= 3i \; .
\end{aligned}
$$

1. Compute the following:
   (a) $z_1 + z_2$
   (b) $z_1 z_2$
   (c) $z_2 - z_3$
   (d) z_1^2
   (e) $z_3 z_1$

2. Convert the following into polar form:
   (a) z_1
   (b) z_2
   (c) z_3

3. Consider the following complex numbers:

$$
\begin{aligned}
z_4 &= 2e^{i\pi} \; , \\
z_5 &= e^{i\frac{\pi}{2}} \; ,
\end{aligned}
$$

   (a) Convert z_4 to regular (non-polar) form
   (b) Convert z_5 to regular (non-polar) form
   (c) Calculate $z_4^* z_4$
   (d) Calculate $z_5^* z_5$

4. **Optional:** Use complex exponentials to derive formuae for $\cos(3x)$ and $\sin(3x)$.

5. **(Optional)** (from p. 369 of Arfkin and Weber, Mathematical Methods for Physicists, fourth edition, Academic Press, 1995.) Prove that:

   (a)
   $$\sum_{n=0}^{N-1} \cos nx = \frac{\sin N(x/2)}{\sin x/2} \cos(N-1)\frac{x}{2} \ , \tag{1}$$

   and

   (b)
   $$\sum_{n=1}^{N-1} \sin nx = \frac{\sin N(x/2)}{\sin x/2} \sin(N-1)\frac{x}{2} \ . \tag{2}$$

   Apparently these sum are used in the analysis of multiple-slit diffraction patterns.

   To do this, you'll then need to use the following result about geometric series:

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \ , \tag{3}$$

for $-1 < r < 1$.