Physics III
Homework Three
Due Friday 13 April, 2007

1. Problem 5 from Chapter 5 of Styer.
2. You have 30 fish in 30 tanks. You want to test and see if the fish, if put together, will fight with each other or get along harmoniously. To do so, you want to put a pair of fish in a special tank and monitor them for a day. Your colleague wants to pair up the 30 fish in each possible way. How many days will it take to complete this experiment? (You only have one of the special observation tanks.)
3. To better understand fish behavior, someone suggests that you should put three fish in the observation tank and see if they all get along, they all fight, or two gang up on one. How long would it take to try every possible trio of 30 fish?
4. You have 15 CDs.
   (a) How many different ways can you arrange them on a shelf?
   (b) If you put the CDs on the shelf at random, what is the probability that they end up in alphabetical order?
5. Suppose you have a box. Inside the box are 13 pieces of paper. One piece of paper has the number 1 written on it, another has the number 2 on it, and so on, up to 13. In order, you grab five pieces of paper from the box.
   (a) How many different 5-number sequences are possible?
   (b) What is the probability that you select the sequence $2, 4, 6, 8, 10$?
   (c) What is the probability that you choose five consecutive numbers?
6. After an in vitro fertilization procedure (IVF) four fertilized eggs are placed in the mother's uterus. Assume that each egg has one chance in ten of implanting successfully, and that this is independent of whether any of the others implants. What is the probability that none of the eggs will implant? That a single egg will implant? That there will be twins? That there will be triplets? That there will be quadruplets? That there will be quintuplets? (From `http://www.math.lsa.umich.edu/~hochster/425/ec2.html`.)

Optional problems. If you want some math challenges, these should give you something fun to work on. If you've had some probability before, I definitely suggest trying at least one or two of these. There is no due date for these problems.

1. A cube that is painted blue is cut into 64 equal cubes. What is the probability P_n that a little cube picked at random has n painted faces, where $n = 0, 1, 2, 3$? (From Garrod, *Statistical Mechanics*, Oxford University Press. 1995.)
2. Ten people throw dice, once per minute, at ten tables. When any person throws 12 he or she leaves. What is the probability that anyone will be left after one hour? (From Garrod, *Statistical Mechanics*, Oxford University Press. 1995.)
3. Suppose that your chance of being in a plane crash on a given flight is one in a million. You are a frequent flier and you fly one million times. What is the probability that you will be in at least one crash? (Assume that the flights are independent events.) For full credit you should observe that the answer can be expressed, almost but not exactly, in terms of one of the fundamental constants of mathematics. Based on a problem from `http://www.math.lsa.umich.edu/~hochster/425/ec3.html`.
4. (Note: this is more difficult than the other problems. Some differential calculus is necessary.) The new breakfast cereal, Millenios, consists of pieces in the two shapes 0 and 2. Thus, a spoonful of these pieces might contain a 2 and the three 0's needed to spell 2000. Suppose that a spoonful of n Millenios is obtained from a machine that produces, independently and randomly, a 2 with probability p and a 0 with probability $1 - p$. (This situation approximates the selection of a spoonful of n pieces from a large box full of Millenios in which the ratio of 2's to 0's is $p : 1 - p$.) (Problem 679 from the *College Journal of Mathematics*.)

   (a) For $n = 4$, what value of p maximizes the probability that the spoonful will contain the 2 and the three 0's needed to spell 2000?
   (b) For arbitrary $n >= 4$, let p_n be the value of p that maximizes the probability that a spoonful of n pieces will contain at least one 2 and at least three 0's. Determine p_n as a function of n.
   (c) Find the limit of p_n from part (2) as n goes to infinity. (It is obvious that for an p satisfying $0 < p < 1$, as n goes to infinity the probability that a spoonful of n pieces will contain at least one 2 and at least three 0's will approach 1.)