Physics III
Homework Four
Due Friday 23 April, 2010

Consider the following complex numbers:

$$
\begin{aligned}
z_1 &= 1 - i \; , \\
z_2 &= -2 \; , \\
z_3 &= 3i \; , \\
z_4 &= 2e^{i\pi} \; , \\
z_5 &= e^{i\frac{\pi}{2}} \; ,
\end{aligned}
$$

1. Compute the following:
   (a) $z_1 + z_2$
   (b) $z_1 z_2$
   (c) $z_2 - z_3$
   (d) z_1^2
   (e) $z_3 z_1$
2. Computer the following:
   (a) $z_1^* z_1$
   (b) $z_2^* z_2$
   (c) $z_3^* z_3$
   (d) $z_4^* z_4$
   (e) $z_5^* z_5$
3. Convert the following into polar form:
   (a) z_1
   (b) z_2
   (c) z_3
4. Consider the following complex numbers:
   (a) Convert z_4 to regular (non-polar) form
   (b) Convert z_5 to regular (non-polar) form
5. McIntyre, et al, Problem 1.1 part a.
6. **Optional:** Use complex exponentials to derive "triple angle formulas." That is, determine expressions for $\cos(3x)$ and $\sin(3x)$ in terms of $\cos(x)$ and $\sin(x)$.

7. **Optional:** Here is another handy use of Euler's formula,

$$e^{ix} = \cos(x) + i\sin(x) \ . \tag{1}$$

Consider the following integral:

$$\int e^{ax}\sin(bx)\,dx \ . \tag{2}$$

Ordinarily, you would do this integral using integration by parts. But there is another way to do it. Re-write the $\sin(bx)$ term in the integrand using Euler's formula. I.e.,

$$\sin(bx) = \Im e^{ibx} \ , \tag{3}$$

where $\Im$ means "imaginary part of." You have now converted the integral into something involving only exponentials. Do the integral and you will get an algebraic expression. Solve for the imaginary part of this expression, and you'll have the answer to the integral of Eq. (2). To do so, you'll need to use Euler's formula in reverse, and will also need to get rid of any i's in the denominator of any fractions.

8. **Optional but recommended for those who have had calculus.** Here is a clever way to prove/verify Euler's formula. Define the following function:

$$g(x) = (\cos x + i\sin x)\,e^{-ix} \ . \tag{4}$$

   (a) Show that $\frac{dg}{dx} = 0$. This means that $g(x)$ is a constant function. Thus, it is the same for all values of x.

   (b) Show that $g(0) = 1$.

   (c) Since $g(x)$ is constant, it must be that $g(x) = g(0)$. Use this fact to derive Euler's formula.