Homework 5

1. Problem 1.1 from McIntyre.
2. Problem 1.3 from McIntyre.
3. Problem 1.5 from McIntyre.
4. Consider the following ket:
   $$|\psi\rangle = a(\,2i|+\rangle - i|-\rangle\,)\,. \tag{1}$$
   Solve for the a that normalizes $|\psi\rangle$.
5. Using the normalized $|\psi\rangle$ from the previous problem, answer the following questions:
   (a) If the atom is in state $|\psi\rangle$, what is the probability of measuring $S_z = 1$?
   (b) If the atom is in state $|\psi\rangle$, what is the probability of measuring $S_z = -1$?
   (c) Do the probabilities add up to 1?
6. Suppose an atom is in the $|-\rangle_x$ state. What is the probability of obtaining a + if S_z is measured? (Show how this number arises from the bras and kets, don't just cite the experimental result.)
7. **(Optional)** Using the normalized $|\psi\rangle$ from the previous problem, answer the following questions:
   (a) If the atom is in state $|\psi\rangle$, what is the probability of measuring $S_x = 1$?
   (b) If the atom is in state $|\psi\rangle$, what is the probability of measuring $S_x = -1$?
   (c) Do the probabilities add up to 1?
8. **(Optional)**
   (a) Use complex exponentials to derive formulae for $\cos 3x$ and $\sin 3x$.
   (b) Use complex exponentials and the binomial theorm to derive general expressions for $\cos nx$ and $\sin nx$, where n is a positive integer.

9. **(Optional)** (from p. 369 of Arfkin and Weber, Mathematical Methods for Physicists, fourth edition, Academic Press, 1995.) Prove that:

   (a)
   $$\sum_{n=0}^{N-1} \cos nx \;=\; \frac{\sin N(x/2)}{\sin x/2} \cos(N-1)\frac{x}{2} \;, \tag{2}$$
   and

   (b)
   $$\sum_{n=1}^{N-1} \sin nx \;=\; \frac{\sin N(x/2)}{\sin x/2} \sin(N-1)\frac{x}{2} \;. \tag{3}$$
   Apparently these sum are used in the analysis of multiple-slit diffraction patterns.

10. **(Optional)** (from p. 369 of Arfkin and Weber, Mathematical Methods for Physicists, fourth edition, Academic Press, 1995.) For $-1 < p < 1$, prove that

    (a)
    $$\sum_{n=0}^{\infty} \;=\; p^n \cos nx \;=\; \frac{1 - p\cos x}{1 - 2p\cos x + p^2} \;, \tag{4}$$
    and,

    (b)
    $$\sum_{n=0}^{\infty} p^n \sin nx \;=\; \frac{p \sin x}{1 - 2p\cos x + p^2} \;. \tag{5}$$
    These sums apparently are used in the theory of the Fabry-Perot interferometer.

Hints for the last two:

1. Stay calm. The expressions are large, but I don't think they require any math that's too fantastic or unthinkable.

2. For each, combine the two sums using complex exponentials: $e^{ix} = \cos x + i \sin x$.

3. You'll then need to use the following result about geometric series:
   $$\sum_{n=1}^{\infty} ar^{n-1} \;=\; \frac{a}{1-r} \;, \tag{6}$$
   for $-1 < r < 1$.