Direct Proofs
&
First Steps into Number Theory
Proofs & Mathematical Structures
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We will take as axiomatic that the integers are *closed* under addition, subtraction, and multiplication. This means that if you add, subtract, or multiply a pair of integers, the answer is another integer.

Definitions are important:

1. An *even* integer n is one which:

2. An *odd* integer m is one which:

1. **Proposition:** The sum of two even integers is even.

2. **Proposition:** The sum of two odd integers is even.

3. **Proposition:** If n is an odd integer, then n^2 is an odd integer.

4. **Proposition:** If n is an integer, then $n^2 + n + 6$ is even.

5. **Proposition:** The product of two odd integers is odd.

6. **Proposition:** The product of an even integer and an odd integer is even.