

# Some Number Theory Practice

## Proofs & Mathematical Structures

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1. Prove that for all  $n \in \mathbb{Z}$ , either  $n^2 \equiv 0 \pmod{4}$  or  $n^2 \equiv 1 \pmod{4}$ .
2. Prove that  $n$  is even if and only if<sup>1</sup>  $n^2$  is even.
3. Let  $a, b \in \mathbb{Z}$  be positive and  $d = \gcd(a, b)$ . Prove that

$$\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1. \quad (1)$$

4. Prove that  $m \equiv n \pmod{15}$  if and only if  $m \equiv n \pmod{3}$  and  $m \equiv n \pmod{5}$ .

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<sup>1</sup>We'll have to talk about what "if and only if" means.