

Here are some problems for Monday, 9 May, 2023.

1. (This is problem 4.30 from the textbook.) A graph is called a *tree* if it can be drawn so that it branches upwards and none of its branches intersect. Two examples of a tree are shown in Fig. 1. Prove that if a tree has n vertices, then it has $n - 1$ edges.

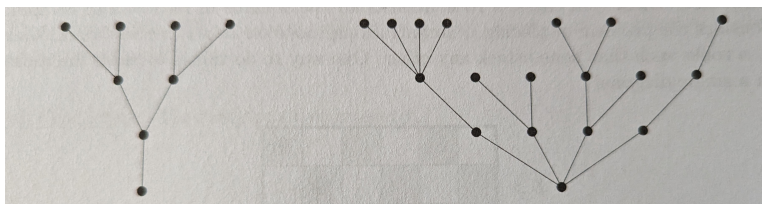


Figure 1: Figure from *Proofs* by Jay Cummings, page 150.

2. Prove that $3^{5n} - 5^{3n}$ is divisible by 59 for any $n \in \mathbb{N}$.
3. **Optional!** Possibly challenging. Possibly interesting. I dunno. Given a positive integer s_1 , let s_2 be the sum of the squares of the digits of s_1 . Let s_3 be the sum of the squares of s_2 , and so on. Prove that for any choice of s_1 , the sequence (s_1, s_2, s_3, \dots) eventually reaches either 1 or 42.