Here are some problems for Wednesday/Thursday May 23/24, 2023. Aim to share solutions to one or two of the problems.

1. Prove that there do not exist three distinct numbers a, b, and c for which: $a+b+c$, ab, ac, bc, and abc are all equal.

2. Prove that if x and y are positive real numbers, then $x+y \geq 2\sqrt{xy}$.

3. Prove that there does not exist an $n \in \mathbb{N}$ for which n^2+n+1 is a perfect square.

4. Prove that if x and y are irrational, then $x+y$ is also irrational.

5. Prove that if x is irrational, then $x+y$ is also irrational.

6. Prove that $\forall x, y \in \mathbb{R}$, such that $x \neq y$,

$$\frac{x}{y} + \frac{y}{x} \leq 2 \; . \tag{1}$$

7. For all real numbers x, with $0 < x < 1$, prove that

$$\frac{1}{x(1-x)} \leq 4 \; . \tag{2}$$

8. Let a, b, c be positive real numbers. Prove that if $ab = c$, then $a \leq \sqrt{c}$ or $b \leq \sqrt{c}$.

9. Suppose $n \in \mathbb{Z}$ is a composite integer. Then n has a prime divisor less than or equal to $\sqrt{n}$.

10. Prove that $x \in \mathbb{R}$ is irrational if and only if it has a different distance from each rational number.

11. Prove that $S_n \notin \mathbb{Z}$ for all $n \geq 2$, where

$$S_n = \sum_{k=1}^{n} \frac{1}{k} \; . \tag{3}$$

The last three problems look interesting and potentially challenging. If more than one group wants to give these a try, that's fine.