Some Exercises for Week 02
Proofs & Mathematical Structures
College of the Atlantic
Due April 14, 2023

Prove the following propositions. The quantities a, b, c, x, y, m, n are all integers.

1. **Proposition:** If $c|a$ and $c|b$, then $c|(a + b)$.

2. **Proposition:** If $c|a$ and $c|b$, then $c|(xa + yb)$.

3. **Proposition:** The product of three consecutive integers is divisible by 3.

4. **Proposition:** Let $m = n(n + 1)(n + 2)$, where $n \in \mathbb{Z}$. Then m is divisible by 6.

5. **Proposition:** Let $a, b, c \in \mathbb{Z}$ satisfy the Pythagorean theorem: $a^2 + b^2 = c^2$. Then either a or b is divisible by 3.

Focus on your proof-writing techniques, especially for the first two. State the proposition that you are proving and include words guiding your reader through your argument.