Introduction to Quantum Mechanics
Homework Four

College of the Atlantic

Due Friday 25 April, 2014

Consider the following complex numbers:

$$\begin{aligned} z_1 &= 1 - i \ , \\ z_2 &= -2 \ , \\ z_3 &= 3 + 3i \ , \\ z_4 &= 2e^{i\pi} \ , \\ z_5 &= e^{i\frac{\pi}{2}} \ , \end{aligned}$$

1. Compute the following, using the non-exponential form for complex numbers:
   (a) $z_1 + z_2$
   (b) $z_2 - z_3$
   (c) $z_3 z_1$
2. Write z_1, z_2, and z_3 as complex exponentials.
3. Compute the following, using the exponential form for complex numbers:
   (a) $z_3 z_1$
   (b) $z_4 z_5$
4. Compute the following, using the exponential form for complex numbers:
   (a) $|z_1|^2$
   (b) $|z_2|^2$
   (c) $|z_3|^2$
   (d) $|z_4|^2$
   (e) $|z_5|^2$
5. Write z_4 and z_5 in non-exponential form. (I.e., as $z = a + ib$.)
6. McIntyre, et al, Problem 1.1 part a.
7. A quantum system is in the state

$$|\psi\rangle = \frac{1}{\sqrt{5}} \left(|+\rangle - 2i|-\rangle \right) \ . \qquad (1)$$

(a) Suppose that S_z is measured. What is the probability of obtaining the result $S_z = +1$?

(b) Suppose that such a measurement is carried out and the result +1 is indeed obtained. What state is the quantum system in, post-measurement?

(c) Now suppose that S_z is measured. What is the probability of obtaining $S_z = -1$?

8. **Optional:** Use complex exponentials to derive "triple angle formulas." That is, determine expressions for $\cos(3x)$ and $\sin(3x)$ in terms of $\cos(x)$ and $\sin(x)$.

9. **Optional:** Here is another handy use of Euler's formula,

$$e^{ix} = \cos(x) + i\sin(x) \ . \tag{2}$$

Consider the following integral:

$$\int e^{ax}\sin(bx)\,dx \ . \tag{3}$$

Ordinarily, you would do this integral using integration by parts. But there is another way to do it. Re-write the $\sin(bx)$ term in the integrand using Euler's formula. I.e.,

$$\sin(bx) = \Im e^{ibx} \ , \tag{4}$$

where $\Im$ means "imaginary part of." You have now converted the integral into something involving only exponentials. Do the integral and you will get an algebraic expression. Solve for the imaginary part of this expression, and you'll have the answer to the integral of Eq. (3). To do so, you'll need to use Euler's formula in reverse, and will also need to get rid of any i's in the denominator of any fractions.

10. **Optional but recommended for those who have had calculus.** Here is a clever way to prove/verify Euler's formula. Define the following function:

$$g(x) = (\cos x + i\sin x)\,e^{-ix} \ . \tag{5}$$

(a) Show that $\frac{dg}{dx} = 0$. This means that $g(x)$ is a constant function. Thus, it is the same for all values of x.

(b) Show that $g(0) = 1$.

(c) Since $g(x)$ is constant, it must be that $g(x) = g(0)$. Use this fact to derive Euler's formula.