Homework One

Thermodynamics

College of the Atlantic

Due Friday, January 10, 2025

All problems are from the textbook, unless otherwise noted.

1. 1.4
2. 1.11
3. 1.12
4. 1.14
5. This is a problem based on question 1.16 from the textbook. The goal is to use Newton's second law ($\vec{F}_{\text{net}} = m\vec{a}$) and the ideal gas law to derive the barometric equation. To do so, consider a slab of air with a thickness of Δz at rest at a height z above the surface of the earth. Denote by M the mass of the air in the slab. Let A be the horizontal area of the slab.

   (a) Use Newton's law to derive an expression for $\frac{dP}{dz}$, the rate at which pressure changes with altitude. *Hints:*

   - The derivative is defined as:

   $$\frac{dP}{dz} = \lim_{\Delta z \to 0} \frac{P(z + \Delta z) - P(z)}{\Delta z} . \tag{1}$$

   - There are three forces acting on the slab.

   (b) Use your answer to the previous problem and the ideal gas law to show that:

   $$\frac{dP}{dz} = -\frac{mg}{kT} P , \tag{2}$$

   where m is the average mass of the air molecules. This equation is known as the barometric equation.

   (c) Show that, assuming that T is constant, the solution to Eq. (2) is given by:

   $$P(z) = P(0) e^{-mgz/kT} , \tag{3}$$

   where $P(0)$ is the pressure at sea level.

   (d) Use Eq. (3) to calculate the pressure, in atmospheres, at the following locations:

   i. Cadillac Mountain
   ii. Katahdin Mountain
   iii. The Zugspitze
   iv. Nevado Huascarán

   Assume that the pressure at sea level is 1 atm.